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MTH 111 Math for Architects Fall 2016, 1–6

87/87

Final Exam, MTH 111, Fall 2016

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QUESTION 1. (12 points).

$f(x) \to f'(x) = 2x$

(i) $\int x e^{(x^2+1)}\, dx =$ $\frac{1}{2}\int 2x e^{(x^2+1)}\, dx = \frac{1}{2} e^{x^2+1} + C$

(ii) $\int \frac{2x+3}{x^2+3x-7}\, dx =$ $\ln|x^2+3x-7| + C$

$f(x) = x^2+3x-7 \to f'(x) = 2x+3$

(iii) $\int (x^2+2)^2\, dx =$ $\int (x^4+4x^2+4)\,dx = \frac{x^5}{5} + \frac{4x^3}{3} + 4x + C$

(iv) $\int \underbrace{(e^x+1)}_{f'(x)}\underbrace{(e^x+x+3)^7}_{f(x)}\, dx =$ $\frac{(e^x+x+3)^8}{8} + C$

QUESTION 2. (12 points). Find y' and do not simplify

(i) $y = e^{(7x^2+5x+1)} + 10x^2 - x + 23$

$' = (14x+5)\, e^{(7x^2+5x+1)} + 20x - 1 + 0$

(ii) $y = (21+2x-4x^3)^5$ $\Rightarrow y' = 5(2-12x^2)(21+2x-4x^3)^4$

(iii) $y = ln[(4x+3)^6(-5x+30)^8]$ $\Rightarrow y = 6\ln(4x+3) + 8\ln(-5x+30)$

$y' = \frac{6(4)}{4x+3} + \frac{8(-5)}{-5x+30} = \frac{24}{(4x+3)} + \frac{(-40)}{(-5x+30)}$

(iv) $x^2y - 3x + xe^y + y^2 + 5y - 200 = 0$

$y' = \frac{-f(x)}{f(y)} = \frac{-[2xy - 3 + e^y]}{x^2 + xe^y + 2y + 5}$

QUESTION 3. (8 points). Let $Q = (2,6)$, $A = (-4,6)$. Find a point B on the line $y = -3$ such that $|QB| + |AB|$ is minimum.

by $y=-3$: $Q' = (2,-12)$

$m\, Q'A = \dfrac{6+12}{-4-2} = \dfrac{18}{-6} = \boxed{-3}$

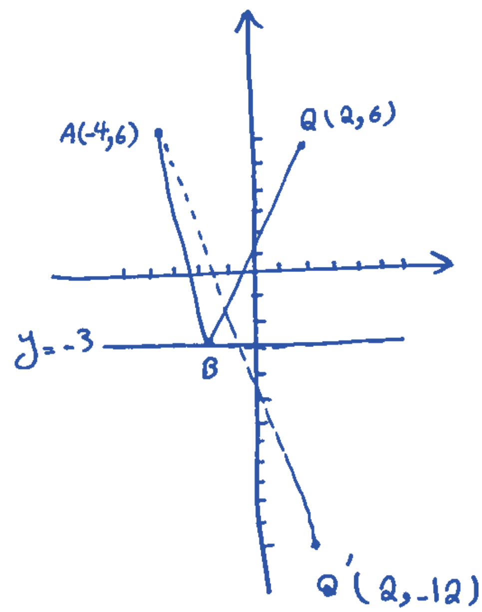
QA: $y = mx + b$

$6 = -3(-4) + b \Rightarrow \boxed{b = -6}$

QA: $\boxed{y = -3x - 6}$

$-3 = -3x - 6 \Rightarrow 3x = -6 + 3 \Rightarrow \boxed{x = -1}$

$\boxed{B = (-1,-3)}$



QUESTION 4. (8 points). Find the area of the region bounded by $f(x) = \sqrt{x} - 2$, $x = 0$ and $x = 9$.

$f(x) = x^{\frac{1}{2}}$

$\displaystyle\int_{x=0}^{x=9} \sqrt{x} - 2 = \left|\int_0^4 (\sqrt{x} - 2)\,dx\right| + \int_4^9 (\sqrt{x} - 2)\,dx =$

$\frac{2x^{\frac{3}{2}}}{3}$

$-\left(\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2x + C \Big|_{x=0}^{4}\right) + \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2x + C \Big|_{x=4}^{x=9} =$

$-\left(\left(\left(\frac{8}{3}\right)^{\frac{3}{2}} - 8 + C\right) - (0 - 0 + C)\right) + \left(\left(\frac{18^{\frac{3}{2}}}{3} - 18 + C\right) - \left(\frac{8^{\frac{3}{2}}}{3} - 8 + C\right)\right) =$

$\left(\frac{-8}{3}\right)^{\frac{3}{2}} + 8 + \left(\frac{18}{3}\right)^{\frac{3}{2}} - 18 - \left(\frac{8}{3}\right)^{\frac{3}{2}} + 8 = \left(\frac{2}{3}\right)^{\frac{3}{2}} - 2 = \frac{\sqrt{8} - 6}{3} = \frac{-3.17}{3}$ X

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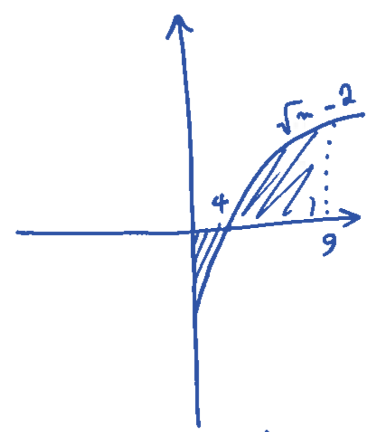
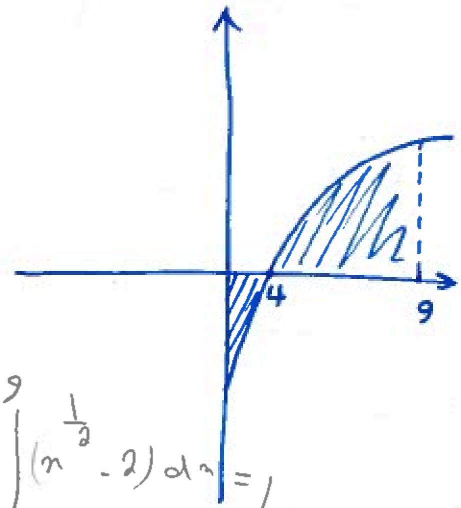


$y = \sqrt{x} - 2$ and $x = (0, 9)$

$\to x^{\frac{1}{2}} - 2$



$$\left| \int_0^4 (x^{\frac{1}{2}} - 2)\,dx \right| + \int_4^9 (x^{\frac{1}{2}} - 2)\,dx =$$

$$-\int_0^4 (x^{\frac{1}{2}} - 2)\,dx + \int_4^9 (x^{\frac{1}{2}} - 2)\,dx = \int_4^0 (x^{\frac{1}{2}} - 2)\,dx + \int_4^9 (x^{\frac{1}{2}} - 2)\,dx =$$

$$\left(\frac{2x^{\frac{3}{2}}}{3} - 2x\right)\Bigg|_{x=4}^{0} + \left(\frac{2x^{\frac{3}{2}}}{3} - 2x\right)\Bigg|_{x=4}^{9} = \left((0+c) - \left(\frac{2.4^{\frac{3}{2}}}{3} - 8\right)\right) + \left(\left(\frac{2 \times 9^{\frac{3}{2}}}{3} - 18\right) - \left(\frac{2.4^{\frac{3}{2}}}{3} - 8\right)\right) =$$

$$\left(\frac{-16}{3} + 8\right) + \left(18 - 18 - \frac{16}{3} + 8\right) = \frac{-32}{3} + 16 = \frac{48 - 32}{3} = \boxed{\frac{16}{3} \text{ unit}^2}$$

QUESTION 5. (4 points). For what values of x does the tangent line to the curve $y = 2e^{(2x-1)} - 8x + 2$ have slope equal four? tangent line $y = mx + b$

$y' = 4e^{(2x-1)} - 8 \longrightarrow 4 = 4e^{(2x-1)} - 8 \Rightarrow 12 = 4e^{(2x-1)} \Rightarrow 3 = e^{(2x-1)} \Rightarrow$

$\ln 3 = \ln e^{(2x-1)} \Rightarrow \ln 3 = (2x-1)\cancel{\ln e} \Rightarrow 1.09 = 2x - 1 \Rightarrow 2.09 = 2x \Rightarrow x = \frac{2.09}{2} =$

$\boxed{1.045}$

QUESTION 6. (8 points). The plane $P_1 : x + y - 2z = 2$ intersects the plane $P_2 : -x + y + 2z = 4$ in a line L. Find a parametric equations of L.

$P_1 : x + y - 2z = 2 \Rightarrow L_1 = \langle 1, 1, -2 \rangle$

$P_2 : -x + y + 2z = 4 \Rightarrow L_2 = \langle -1, 1, 2 \rangle$

$$L_1 \times L_2 : \begin{array}{c|ccc|} & i & j & k \\ L_1 & 1 & 1 & -2 \\ L_2 & -1 & 1 & 2 \end{array} = (2+2)\,i - (2-2)\,j + (1+1)\,k = 4i + 2k$$

$\langle 4, 0, 2 \rangle$

$z = 0 \Rightarrow \left\{ \begin{array}{l} x + y = 2 \\ -x + y = 4 \end{array} \right\} \quad 2y = 6 \Rightarrow \boxed{y = 3}$ $\boxed{x = -1}$

$L : \langle 4, 0, 2 \rangle t + (-1, 3, 0) = \langle 4t - 1, 3, 2t \rangle \rightarrow L : \boxed{\begin{array}{l} x = 4t - 1 \\ y = 3 \\ z = 2t \end{array}}$

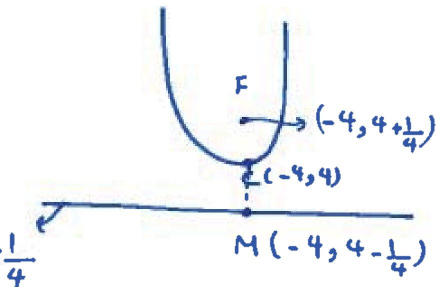
QUESTION 7. (8 points). Given $y = x^2 + 8x + 20$

(i) Roughly, Sketch the graph of the given parabola.

$y = (x+4)^2 - 16 + 20 \Rightarrow y = (x+4)^2 + 4$

$(y-4) = (x+4)^2$

$4d(y - y_0) = (x - x_0)^2$



(ii) What is the directrix line?

$4d = 1 \Rightarrow \boxed{d = \frac{1}{4}}$

$\boxed{C = (-4, 4)}$

$\rightsquigarrow$ directrix line: $\boxed{y = 4 - \frac{1}{4} = \frac{15}{4}}$

(iii) What is the focus?

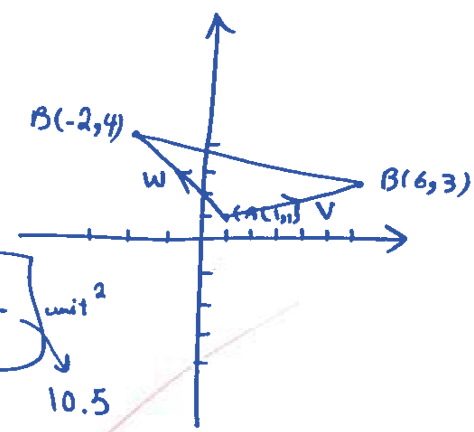
$\boxed{F = \left(-4, 4 + \frac{1}{4}\right)}$

QUESTION 8. (5 points). Given $\overset{A}{(1,1)}, \overset{C}{(6,3)}, \overset{B}{(-2,4)}$ are the vertices of a triangle. Find the area of the triangle.

$A(1,1)$ $\langle -3, 3 \rangle$ $B(-2,4)$

w $\langle 5, 2 \rangle$ v

$C(6,3)$



$\text{area} = \frac{|w \times v|}{2} = \frac{\left| \begin{matrix} -3 & 3 \\ 5 & 2 \end{matrix} \right|}{2} = \frac{|-6-15|}{2} = \frac{|-21|}{2} = \boxed{\frac{21}{2}}$ unit^2 $\to$ 10.5

~~wxv~~

$\text{area} = \frac{v \times w}{2} = \frac{\left| \begin{matrix} 5 & 2 \\ -3 & 3 \end{matrix} \right|}{2} = \frac{15+6}{2} = \frac{21}{2}$

2

* **QUESTION 9. (4 points).** Can we draw the vector $< 4, -5, -2 >$ inside the plane $2x - 6y + 19z = 20$? EXPLAIN

$2x - 6y + 19z - 20 = 0$

a point on plane: $(10, 0, 0)$ = initial point

$(10,0,0) + \langle 4, -5, -2 \rangle = \underbrace{(14, -5, -2)}_{\text{Terminal}} \rightsquigarrow$ check if the point still on the plane.

$28 + 30 + \overset{-38}{(-2)(19)} - 20 = \boxed{0}$ ✓ $\to$ yes we can draw this vector because the terminal point also exists on the plane.

2

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QUESTION 10. (9 points).

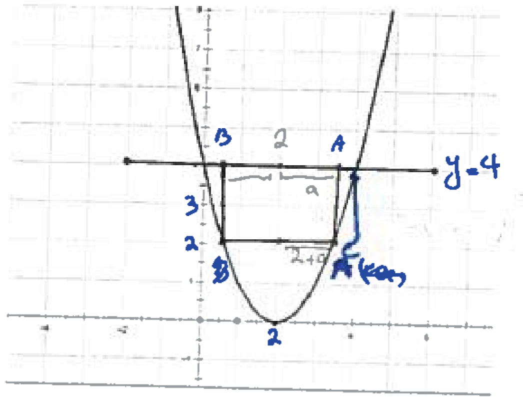

Figure 1. Question: Find the length and the the width of the rectangle as in picture that has maximum area (The curve is $y = x^2 - 4x + 4 = (x-2)^2$ and the horizontal line is y = 4]

$A = (2+a, 4)$
$B = (2-a, 4)$ $AB = 2+a-2+a = \boxed{2a}$

$C = (2+a, (2+a-2)^2) = (2+a, a^2) \longrightarrow AC = \boxed{4-a^2}$

$f(a) = \text{area} = 2a(4-a^2) = 8a - 2a^3 \Rightarrow f'(a) = 8 - 6a^2 \Rightarrow$

max area $\Rightarrow 8 - 6a^2 = 0 \Rightarrow 8 = 6a^2 \Rightarrow 4 = 3a^2 \Rightarrow a^2 = \frac{4}{3} \Rightarrow \boxed{a = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}}$

$\boxed{AB = \frac{4}{\sqrt{3}}}$ $\boxed{AC = 4 - \frac{4}{3} = \frac{12-4}{3} = \frac{8}{3}}$

$\frac{4\sqrt{3}}{3}$

$a(4-a^2) = 8/4$

$4a - a^3$

$4 - 3a^2$

$a = 2/\sqrt{3}$

QUESTION 11. (9 points).

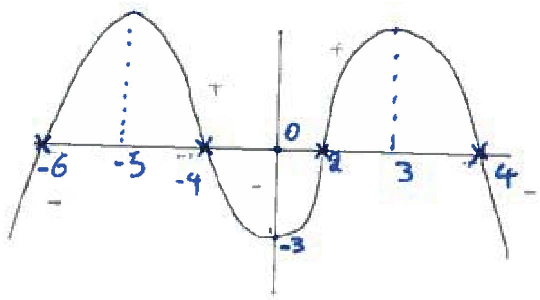

Figure 2. Question: You are looking at the curve of $f'(x)$.

(i) Find all x values where $f(x)$ is maximum.

$x: \boxed{-4, 4}$

(ii) Find all x values where $f(x)$ is minimum.

$x: \boxed{-6, 2}$

(iii) For what values of x does $f(x)$ increase?

$x \in: (-6, -4) \cup (2, 4)$

(iv) For what values of x does $f(x)$ decrease?

$x \in: (-\infty, -6) \cup (-4, 2) \cup (4, +\infty)$

(v) For what values of x do the slopes of tangent lines are positive?

$x \in (-6, -4) \cup (2, 4)$

(vi) For what values of x do the slopes of normal lines are negative?

$x \in (-6, -4) \cup (2, 4)$

Faculty information

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